

## Accurate *Body Density Index* $BDI_n$ to replace flawed BMI

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**Abstract:** Body Mass Index (BMI) distorts most individual's adiposity, giving the same result for individuals having the same weight and height despite who is flabby and who is lean. A more accurate index of adiposity would improve clinicians' assessment of health risks and guidance for weight control while avoiding patient anxiety over self-image. Should BMI be superseded? This paper proposes  $BDI_n$  ("BDI-sub-n"), *Body Density Index normalized* within entrenched BMI categories, using size measurements that take only a minute more than BMI to make, but optimized using a bio-engineering model for body volume and an online data repository & calculator.

### History of BMI and its well-known issues

Two centuries from conception and decades in clinical practice, Body Mass Index (BMI) distorts individual body fat.<sup>1 2 3 4 5</sup> It fails to register leanness, using only two measurements, weight  $w$  & height  $h$ :  $BMI = w/h^2$  [1]. Absent the volume of each body shape, BMI ignores the density proportions of fat, muscle, & bone.<sup>6</sup> *BMI is the same for any two people with the same weight & height despite which is flabby and which is lean.* The AMA advises "*BMI can be inaccurate and misleading.*"<sup>a</sup> The National Institutes of Health (NIH) concedes "*BMI is a poor indicator of percent of body fat.*"<sup>b 7</sup> NPR calls BMI "bogus;" The NY Times calls BMI "a scam."

A century after its invention for Belgium's male population, but specifying it was not reversible for individuals,<sup>8</sup> its formula was usurped as an *individual* metric named BMI. Soon suspiciously round-numbered fatness categories were added to differentiate levels of leanness (muscle), but not of women, various races, or fat location (healthier *pear* v. dangerous *apple*.)<sup>9</sup> Physicians using BMI may well err in assessing a patient's health risks, or in giving dietary advice.<sup>10</sup> Being told one's BMI can trigger anxiety re body self-image that distracts patients from heeding a doctor's advice.

In 1832, renown statistician Adolphe Quetelet annually tracked the Belgian population of males'  $w/h^2$ , where his very simple math formula's inconsistencies simply averaged out by the *law of large numbers*. In 1972, Dr. Ancel Keys coined  $w/h^2$  as "BMI."<sup>c 11</sup> Keys was criticized for 50+ years for omitting data that disagreed with his premise, and for cataloging only males. MetLife adopted BMI to set progressive life insurance rates that inflate over time.<sup>d 12</sup>

BMI captured the world of medicine in 1973, first by the Royal College of Physicians. Some believed in error that BMI was the % body fat. Then in 1997 the WHO established obesity categories<sup>13</sup>

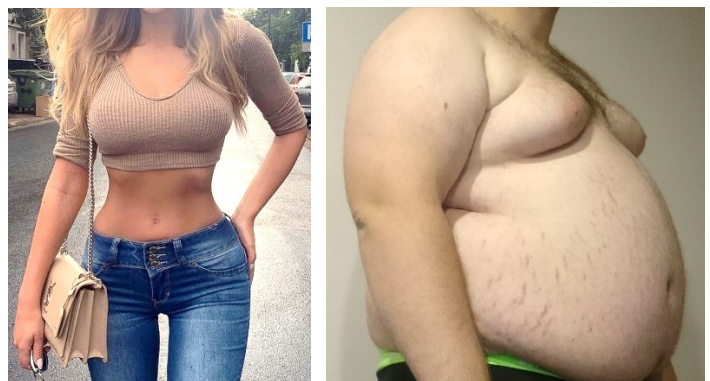
for BMI, anecdotally observed round numbers, and applying only to white males, with *Normal* ranging 18.5~24.9 (21.5  $\pm$  3). With varying bone & muscle densities, there has been no appropriate Normal for women, nor for different races' range of body shapes.

This paper introduces  $BDI_n$  (BDI-sub-n), an accurate individual Body Density Index *normalized* to fit BMI's familiar categories. In sum,  $BDI_n$  measures adiposity by overall body density for all adults including women, Asians, and African Americans; BMI does not.

### BMI as applied to individuals is mathematically disproved

Fat is less dense than muscle, bone, & water. So in water a fat person floats; a lean person sinks. If a person's weight increase is due to fat, BMI rises. But if a person's weight increase is muscle, instead of falling BMI still rises, reporting lean athletes as "obese." Improperly applied to individuals, Quetelet's 1832 formula causes this reciprocity, magnifying BMI's error randomly by individual.

This paper quantifies BMI's distortions that err by between -8.4 to +11.0 points.<sup>e</sup> *Is it time to replace BMI?* Many recognize BMI's flaws, but an Oxford mathematician's "New BMI" still uses only  $w$  and  $h$ .<sup>f 14</sup> Monthly personal Bioelectric Impedance Analysis (BIA) may track relative fat v. muscle loss or gain consistently enough, but it is absolutely inaccurate and unstable with hydration & exercise, thus is unsuited for infrequent clinical assessment. Mayo Clinic scanning is too costly for clinical practice. Proposed  $BDI_n$  requires 4 additional size measurements that take a minute.



**L:** Underweight in BMI of 18.2 but healthy in  $BDI_n$  of 20.6 - Google image.

**R:** Morbidly obese male of BMI 52.9 and waist:hips >1.0 - Wikipedia.

**Below:** Body Composition should be differentiated by gender and race.

|               | Men | Women |
|---------------|-----|-------|
| Muscle        | 45% | 36%   |
| Essential Fat | 3%  | 12%   |
| Storage Fat   | 12% | 15%   |
| Bones         | 15% | 12%   |
| Others        | 25% | 25%   |

<sup>a</sup> AMA 2023 - <https://www.ama-assn.org/delivering-care/public-health/ama-use-bmi-alone-imperfect-clinical-measure>

<sup>b</sup> NIH 2015 - <https://pmc.ncbi.nlm.nih.gov/articles/PMC4890841/>

<sup>c</sup> Ancel Keys-Seven Countries Study - <https://pubmed.ncbi.nlm.nih.gov/31338844/>; <https://www.scienceofeds.org/wp-content/uploads/pekar-ims-bmi.pdf> (but only of males).

<sup>d</sup> Keys lobbied for MetLife to abandon its 1943 *Weight for Height* and adopt his "BMI" (Quetelet's 1832  $w/h^2$ ) - <http://www.assessmentpsychology.com/metlife.htm>

<sup>e</sup> A 19.4-point spread; later the same point spread for BMI cf.  $BDI_n$  of -6 to +13.4.

<sup>f</sup> N Trefethen, Prof Numerical Analysis, Univ. of Oxford, "New BMI" =  $1.3 \cdot w/h^2 \cdot 2.5$ ; C Scamahorn "BFATI" [Old  $BDI_n$ ] 2006 =  $W(w+b)/h^3 \text{ kg} \cdot \text{mm}/\text{mm}^3$  [-kg/mm<sup>3</sup>??]

Shown in **Fig.1**, BMI though statistically brilliant for averaging a population (**dashed orange trendline**), is not for any individual.<sup>8 15</sup> Nor for most Asians, African Americans, children, women. Do we expect the *ideal* for any individual to conform to an average?

BMI describes as “normal 21.5” the index of a healthy male, i.e. an 1830’s Belgian of 57.73kg (127lb) and 1.638m (64.5in). Normal

$$\text{BMI} = w/h^2 = 57.73 / 1.638^2 = \mathbf{21.52} \text{ ostensibly kg/m}^2 \text{ [2]}$$

or in inch-pound units common in North America and elsewhere

$$\text{BMI} = w/h^2 = 703 \times 127\text{lb} / 64.5\text{in}^2 = \mathbf{21.46} \text{ kg/m}^2 \text{ [3].}$$

BMI in  $\text{kg/m}^2$  is not density, which would be  $\text{kg/m}^3$ , or  $\text{g/cm}^3$ . Density indicates body composition as mass per unit volume, in earth’s gravity  $\rho = w/V$ . Viewed by the *factor-label* method and simplifying V to a cube,  $V = hhh$ ,  $\rho = w/hhh$ . BMI  $= w/hh$ . With its sole shape factor  $h^2$ , BMI cannot represent body leanness.<sup>g 16</sup>

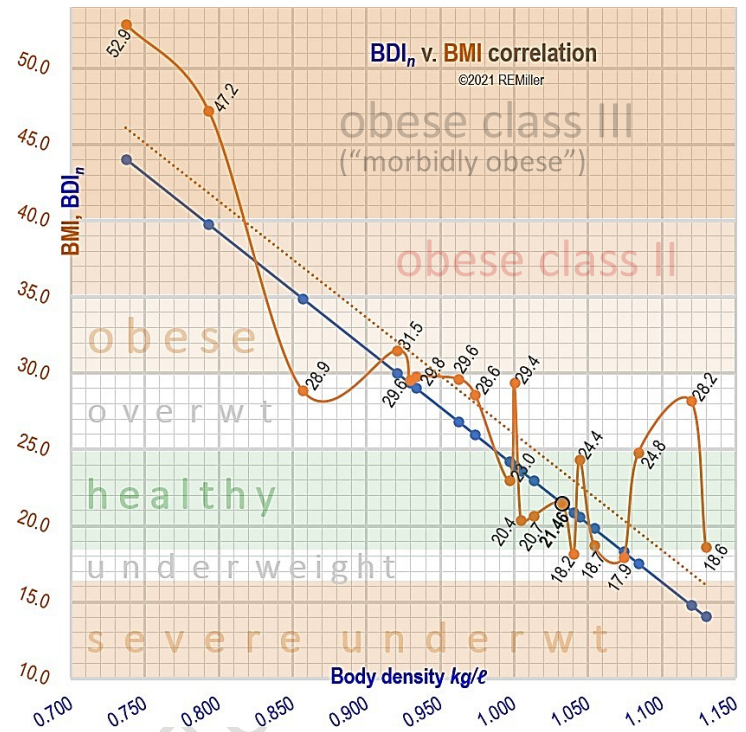
In clinical examinations, BMI is expressed as a dimensionless “index” clinicians look up in a printed table, or read as calculated on a computerized form. Proposed  $\text{BDI}_n$  (“BDI sub n”) is based on accurate body V that divided into weight w equals accurate body density BD. Then *normalized* to familiar BMI categories as  $\text{BDI}_n$ .<sup>h</sup>

**Fig.1** plots the small “population” in **Table.1** spanning individual body densities 0.737 to 1.130  $\text{g/cm}^3$  (Normal is 1.033  $\text{g/cm}^3$ ) and BMI\( $\text{BDI}_n$ ) indices ranging 14 to 53 (Normal is bold 21.46). Shading delineates arbitrary categories “severe underweight” <16.5 to “morbidly obese” >40.<sup>i</sup> Closer to ~21.5 would imply a healthier male population with fewer people obese or underweight.

Plotted by body density BD on the horizontal axis, an individual generates a pair of adiposity index points, an **orange** dot for BMI and a **blue** dot for  $\text{BDI}_n$ . Each pair’s vertical separation is BMI’s error for that individual. The 20 individuals’ BMIs (**solid orange line**) career about their **dashed orange trendline** between -8.4 and +11.0 points on the vertical axis, swinging -29% and +39% from to BMI’s own *least squares* trendline. Were it accurate, BMI’s individual datapoints would be closer to its trendline.

$\text{BDI}_n$  for all individuals are inherently linear, not distorting their obesity.  **$\text{BDI}_n$  in blue** is *normalized* to parallel BMI’s trendline, and to contain its Normal of 21.46.  $\text{BDI}_n$  datapoints directly below or above corresponding BMI points differ by between -6 and +13.4 points, implying individual corrections between +17% and -90%.

$\text{BDI}_n$  is refined with each added dataset, but we do not expect it to fall on BMI’s trendline. If BMI’s arbitrary categories are to be improved on an objective basis, then the form  $\text{BDI}_x$  can be re-normalized to plot consistently within them. The key is a more accurate body density  $\text{BD} = w/V_{\text{body}}$  from a truer body volume  $V_{\text{body}}$ .



**Fig.1** - BMI by individual —●— is often far from its own trendline ....  $\text{BDI}_n$  —●— based on body density is linear (undistorted). Normalized to BMI categories,  $\text{BDI}_n$  is a truer individual fatness index. BMI’s error for any individual’s pair of ● and ● above or below is calculated in **Table.1** under “differ” & “differ%.”

#### A measure of adiposity relies on an accurate body volume $V_{\text{body}}$

An index of body fat derives from density,  $\rho = m/V$ . In space, a mass  $m$  just sits there, but on earth, gravity gives  $m$  the force of weight  $w$ . Then  $w/V_{\text{body}}$  is body density  $\text{BD}$  for any male, female, any race, lean\athletic, or aged body shapes.<sup>j</sup> An Adiposity index depends on an accurate estimate of  $V_{\text{body}}$  giving a more accurate result than BMI’s  $h^2$ . And still more accurate with more data.

We measure body volume  $V_{\text{body}}$  not by dunking or an expensive scanner, but by a bio-engineering model. In Fig.2 L, the simplified model calculates  $V_{\text{body}}$  from a minimum number of five (5) size measures: the usual overall height  $h$ , plus an additional four: sitting height  $h_{\text{sit}}$ , waist depth  $d$ , and perimeters waist  $p_w$  and hips  $p_h$  that with practice take only one minute more to measure.

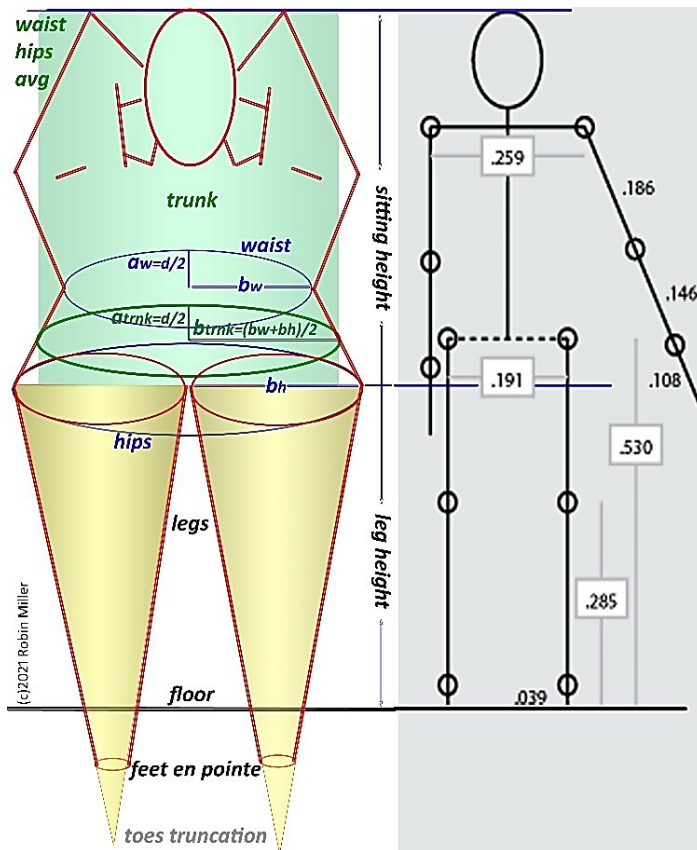
Simply put, the  $V_{\text{body}}$  model is an elliptic cylinder for the upper body atop two truncated cones for legs.  $V_{\text{body}} = V_{\text{trunk}} + V_{\text{legs}(2)}$  [4]. **Fig.2L** shows the cylinder integrating the torso, head, and arms by envisioning clasped hands behind head and tucked elbows in and up, even with the crown. Arm volume is assumed proportional to the trunk. As brain density is higher than average BD, the head is in effect a larger volume, dealt with after a 1<sup>st</sup> approximation. The mathematical derivation of  $V_{\text{body}}$  modeled in **Fig.2L** is in **Appx.C**.

<sup>g</sup> A Journal of the American Heart Association 2018 study finds girth measures a better indicator of heart attack risk than BMI. **Table.1** uses 0.78 ratio of waist:hips.

<sup>h</sup>  $\text{BDI}_n$  when normalized to BMI, otherwise  $\text{BDI}_x$  is extensible for any calibration.

<sup>i</sup> **Table.1** does not represent the US pop., which per CDC is on average borderline obese, but shows how BMI wrongly categorizes both the unhealthy and healthy.

<sup>j</sup> Density of water by definition equals 1.000  $\text{g/cm}^3$  (1.000  $\text{kg/l}$ ), or ~0.036  $\text{lb/in}^3$ .



**Fig.2 L** –  $BDI_n$  is based on a sufficiently accurate model of body volume  $V_{body}$ , an elliptical cylinder + two cones, from five measurements, the math in **Appx.C**. **R** - Mean anthropometrics ratios of Drillis & Contini, segmented at joints.<sup>k 17</sup>

From 1970 to 2019, U.S. mean age rose from 28yr to 38yr. Per **Appx B**, Americans today average 50lb heavier and 8 points higher in BMI averaged. Taller by 2in, one can have a leaner look, yet we still use 1997 BMI fatness categories, with “normal” equivalent to a 1830s Belgian of 127lb & 5ft 4½in. BMI and its categories do not register fat v. muscle v. bone, gender, age, or waist to hips ratio.

Humans only  $\frac{1}{13}$  cubic meters in mean volume suggests that a body density **BD** in metric tons per cubic meter is hardly intuitive. A more understandable body density **BD** is in kg/liter  $\ell$  ( $=g/cm^3$ ):

$$BD \approx Cw/V_{body} \text{ kg}/\ell, \text{ g}/cm^3, \text{ lb}/in^3 \text{ (after } V_{body} \text{ from Appx C)} \quad [13^1]$$

for  $C=1000cm^3/\ell$  for metric units or  $=27.68 \text{ in}^3/lb/kg\ell$  ( $61in^3/\ell \div 2.2046lb/kg$ ) for Imperial inputs. The complete body density is

$$BD \approx Cw/(\Delta h_{sit}\pi d(0.1904p_h + \sqrt{(p_w^2/(2\pi^2) - (d/2)^2)})/4 + 0.0277p_h^2(h - h_{sit})) \text{ kg}/\ell \text{ (g}/cm^3) \text{ or } lb/in^3 \quad [14]$$

where  $w$  is weight in  $kg$  or  $lb$ , and  $p$ 's and  $h$ 's are in  $cm$  or  $inches$ . (For Imperial inputs in  $lb$  &  $in$ , **BD** is converted to metric  $kg/\ell$ .) Now to transform **BD** to an equally sensitive index  $BDI_n$  that is as familiar to health providers & patients as **BMI**, but more accurate for more properly advising patients about adiposity's health risks.

### Accurate Body Density Index $BDI_n$ normalized to BMI categories

After 50+yr, **BMI** is entrenched in the healthcare community, along with its round-number categories.  $BDI_n$  normalizes **BD** to **BMI**'s trend line slope & intercept,  $y = Mx + N$ , and scales **BD** within bounds of 0% muscle (impossible) and 0% fat (2% is possible). Fat is  $\sim 0.9 \text{ kg}/\ell$ ; muscle  $\sim 1.1 \text{ kg}/\ell$ ; bone up to  $1.38 \text{ kg}/\ell$ .

<sup>m</sup> Humans average  $\sim 15\%$  bone;  $\sim 85\%$  fat, muscle & watery fluids.

<sup>18</sup> So body density **BD** falls below the weighted inequality

$$BD < 85\% \cdot 1.1 + 15\% \cdot 1.38 \quad BD < 1.142 \text{ kg}/\ell \quad [15]$$

With the **BD** limited to 1.142, **BMI**'s trendline in **Fig.1** slopes by  $M=76.42$  refers to a Normal of  $21.5 = M \cdot (1.142 - BD) + N$  that solves for  $N=13.12$ ,<sup>n</sup> now as **Body Density Index normalized** (“**BDI-sub-n**”)

$$BDI_n = 76.42 \cdot (1.142 - BD) + 13.12 \text{ (kg}/\ell \text{ normalized index)} \quad [16].$$

The complete expression for “**Body Density Index, normalized, BDI-sub-n**,” is

$$BDI_n \approx N + M \cdot (1.142 - Cw/(\Delta h_{sit}\pi d(0.1904p_h + \sqrt{(p_w^2/(2\pi^2) - (d/2)^2})^{0.5})/4 + 0.0277p_h^2(h - h_{sit}))) \text{ (index normalized to BMI)} \quad [17].$$

Each added data set to the 20 illustrated in **Fig.1** can serve to fine tune normalization slope  $M$  and intercept  $N$ . A scalable form  $BDI_x$  can fit any new and more reasoned adiposity categories by new data varying slope constants  $N$  &  $M$ .  $\Delta$  is a volume model error corrector. The total of six variable size measurements are:  $w$  is weight in  $kg$  or  $lb$ ; in  $cm$  or  $in$ ,  $p$ 's are waist & hips perimeters,  $d$  is trunk depth, the  $h$ 's are standing & sitting heights. Substituting in **bold BMI**'s  $w$  &  $h$  plus four added measurements taking a minute per **Appx.A**,  $BDI_n$

$$= 13.12 + 76.42 \cdot (1.142 - Cw/((0.925 \cdot h_{sit} \cdot 3.14 \cdot d \cdot ((0.1904 \cdot p_h + (\sqrt{(p_w^2/(2 \cdot 3.14^2) - (d/2)^2})^{0.5}})/4) + 0.0277 \cdot p_h^2 \cdot (h - h_{sit}))) \text{ (procedure in Appx.A)} \quad [18]$$

### Examples of calculating $BDI_n$ per **Appx.A** in contrast to **BMI**

**Ex1:** In the appropriate measurement units, calculate using **Appendix A**  $BDI_n$  for: **weight** 127lb, **h** 5ft4½in, **h<sub>sit</sub>** 29in, **waist** 30in, **d** 8.0in, **hips** 34in:

$$13.12 + 76.42 \cdot (1.142 - 27.68 \cdot 127 / ((0.925 \cdot 29 \cdot 3.14 \cdot 8.0 \cdot ((0.1904 \cdot 34 + (\sqrt{(30^2/(2 \cdot 3.14^2) - (8.0/2)^2})^{0.5}})/4) + 0.0277 \cdot 34^2 \cdot (64.5 - 29))) \text{ [copy\&search, per Appx.A]}$$

$$BMI = 21.5 \text{ is “normal,” but } BDI_n \approx 14.96 \text{ (anorexic)}$$

**Ex2:** Find **BDI**: **w**57.6kg, **h**163.8cm, **h<sub>sit</sub>**86cm, **p<sub>w</sub>**86cm, **d**25cm, **p<sub>h</sub>**86cm:

$$13.12 + 76.42 \cdot (1.142 - 1000 \cdot 57.6 / ((0.925 \cdot 86 \cdot 3.14 \cdot 25 \cdot ((0.1904 \cdot 86 + (\sqrt{(86^2/(2 \cdot 3.14^2) - (25/2)^2})^{0.5}})/4) + 0.0244 \cdot 86^2 \cdot (163.8 - 86))) \text{ [copy\&search, per Appx.A]}$$

$$BMI = 21.5 \text{ is “normal,” but } BDI_n \approx 30.18 \text{ (obese)}$$

Note: **Ex1&2** are identical in weight & height for identical **BMI**, but different in **BMI**-ignored waist, depth, & hips that  $BDI_n$  includes.

**Ex3:** Try it. Your own measurements per **Appx.A**. Any surprises?

<sup>k</sup> Drillis & Contini 1966 Body segment parameters #1166.03, New York University

<sup>1</sup> To preserve math continuity, formulae numbers in [ \_ ] continue after **Appx.C**.

<sup>m</sup> ~1.38 Zioupos et al Bone Density <https://doi.org/10.1016/j.jbiomech.2008.03.025>

<sup>n</sup> Anywhere a **BMI** datum is on its trendline is valid to find  $N = BMI_n - M(1.142 - BD_n)$ .

## Results comparing BDI<sub>n</sub> vs. BMI for 20 “individuals”

Table.1 calculates BDI<sub>n</sub> v. BMI for 20 datasets, graphed in Fig.1, including 2021 US Centers for Disease Control & Prevention (CDC) data, and the widely accepted modern body density BD average of 0.985. Each row is an individual, sorted by BD. For illustration, estimated data in green are rounded extrapolations. Additional data sets are needed to confirm BDI<sub>n</sub>'s effectiveness, to fine tune its normalization to BMI categories, or to fine tune those.

For normalizing to BMI's familiar categories, we make BDI<sub>n</sub>'s trendline parallel to BMI's and to contain its Normal 21.5, so data by individual fall on BDI<sub>n</sub>'s trendline. Fig.1 shows BMI swinging erratically among individuals, as they vary from its own trendline

“BMI<sub>VTL%</sub>” by -8.4 to +11.0 points, between -29 and +39%. BMI “differs” from BDI<sub>n</sub> between -6 lower to +13.4 points higher, as calculated under “differ” and “differ%” in Table.1. Not perfect, BDI<sub>n</sub> need only be far better by individual than BMI to cover its cost in practice – making four additional measurements that take only a minute more. For better advice by physicians that helps fulfill their intentions of better health for their patients.

App.D shows two other contenders to replace for BMI: “New BMI” that still uses only *w* & *h*; “BFAT1” with sizes in *mm* that force results into the ballpark. As with BMI, any index not based on body density often shows a healthy person as underweight or obese or v.v., determinations that are less correct than BDI<sub>n</sub>.

Table.1 - Calculated BD and BDI<sub>n</sub> v. BMI for 20 body datasets in yellow s columns (in Imperial units), sorted by increasing body density BD.

| BDI v. BMI (imper.) ©REMILLER210816d                                                                                       |                                      |       |       |                       |      |                     |          |        |         | M <sub>slope</sub> = 76.420 |                      | N <sub>norm</sub> = 13.12 |                     | Δ = 0.9250 |        | 0.712-1.142 = Mx+N-BMI trendline |                    | 1.3w/h <sup>2.5</sup> |               | W(w+b)/h <sup>3</sup> |  | BMI v. BDI |  | BMI v. TrendLn |  | p <sub>w</sub> : p <sub>h</sub> |  |
|----------------------------------------------------------------------------------------------------------------------------|--------------------------------------|-------|-------|-----------------------|------|---------------------|----------|--------|---------|-----------------------------|----------------------|---------------------------|---------------------|------------|--------|----------------------------------|--------------------|-----------------------|---------------|-----------------------|--|------------|--|----------------|--|---------------------------------|--|
| Sbj                                                                                                                        | Catgry                               | w lb  | ΔCDC  | h <sub>stand</sub> in | ΔCDC | h <sub>sit</sub> in | waist in | dep in | hips in | BD                          | BDI <sub>n</sub>     | BMI                       | "New" BMI           | BFAT1mm    | differ | differ%                          | BMI <sub>vTL</sub> | BMI <sub>vTL%</sub>   | ideals ≤ 0.80 |                       |  |            |  |                |  |                                 |  |
| 1                                                                                                                          | ♂Vhvy                                | 401.0 | 230.2 | 73.00                 | 4.0  | 40.00               | 65.00    | 20.0   | 55.00   | 0.737                       | 44.1                 | 52.90                     | 50.5                | 79.8       | 8.8    | 20%                              | 6.5                | 12%                   | 1.18          |                       |  |            |  |                |  |                                 |  |
| 2                                                                                                                          | ♀Vhvy                                | 250.0 | 79.2  | 61.00                 | -2.5 | 34.00               | 55.00    | 13.0   | 53.00   | 0.793                       | 39.8                 | 47.23                     | 49.3                | 76.7       | 7.5    | 19%                              | 5.1                | 11%                   | 1.04          |                       |  |            |  |                |  |                                 |  |
| 3                                                                                                                          | ♂Sr hvy <sup>1</sup>                 | 228.0 | 57.2  | 74.50                 | 5.5  | 35.50               | 46.00    | 13.0   | 42.50   | 0.857                       | 34.9                 | 28.88                     | 27.3                | 31.5       | -6.0   | -17%                             | -8.4               | -29%                  | 1.08          |                       |  |            |  |                |  |                                 |  |
| 4                                                                                                                          | ♀Sr hvy                              | 175.0 | 4.2   | 62.50                 | -1.0 | 32.50               | 40.00    | 10.5   | 42.00   | 0.921                       | 30.0                 | 31.49                     | 32.5                | 37.9       | 1.5    | 5%                               | -0.9               | -3%                   | 0.95          |                       |  |            |  |                |  |                                 |  |
| 5                                                                                                                          | ♂Sr Lg <sup>3</sup>                  | 215.0 | 44.2  | 71.50                 | 2.5  | 35.50               | 41.00    | 12.0   | 42.00   | 0.929                       | 29.4                 | 29.57                     | 28.5                | 31.5       | 0.2    | 1%                               | -2.2               | -7%                   | 0.98          |                       |  |            |  |                |  |                                 |  |
| 6                                                                                                                          | ♀CDC2021                             | 170.8 | 0.0   | 63.50                 | 0.0  | 33.40               | 37.95    | 9.5    | 43.00   | 0.933                       | 29.1                 | 29.78                     | 30.5                | 34.8       | 0.7    | 2%                               | -1.7               | -6%                   | 0.88          |                       |  |            |  |                |  |                                 |  |
| 7                                                                                                                          | ♀&♂CDC                               | 185.3 | 14.5  | 66.30                 | 0.0  | 35.24               | 39.60    | 9.6    | 42.00   | 0.963                       | 26.8                 | 29.63                     | 29.7                | 33.5       | 2.8    | 10%                              | 0.4                | 1%                    | 0.94          |                       |  |            |  |                |  |                                 |  |
| 8                                                                                                                          | ♂Sr Lg <sup>3</sup> die <sup>4</sup> | 208.0 | 37.2  | 71.50                 | 2.5  | 35.50               | 40.00    | 11.0   | 41.00   | 0.974                       | 26.0                 | 28.60                     | 27.6                | 29.7       | 2.6    | 10%                              | 0.3                | 1%                    | 0.98          |                       |  |            |  |                |  |                                 |  |
| 9                                                                                                                          | ♀fit 40\50                           | 138.0 | -32.8 | 65.00                 | 1.5  | 35.00               | 29.00    | 9.0    | 37.00   | 0.997                       | 24.2                 | 22.96                     | 23.2                | 21.4       | -1.2   | -5%                              | -3.6               | -16%                  | 0.78          |                       |  |            |  |                |  |                                 |  |
| 10                                                                                                                         | ♂CDC2021                             | 199.8 | 29.0  | 69.10                 | 0.1  | 37.00               | 40.08    | 9.8    | 41.00   | 1.000                       | 24.0                 | 29.42                     | 28.9                | 31.7       | 5.5    | 23%                              | 3.1                | 10%                   | 0.98          |                       |  |            |  |                |  |                                 |  |
| 11                                                                                                                         | ♂Sr Md                               | 138.0 | -32.8 | 69.00                 | 0.0  | 35.75               | 32.00    | 8.0    | 35.50   | 1.005                       | 23.6                 | 20.38                     | 20.0                | 18.3       | -3.2   | -14%                             | -5.6               | -28%                  | 0.90          |                       |  |            |  |                |  |                                 |  |
| 12                                                                                                                         | ♀Sr xSm <sup>5</sup>                 | 104.0 | -66.8 | 59.50                 | -4.0 | 28.00               | 28.50    | 6.9    | 36.00   | 1.013                       | 23.0                 | 20.65                     | 21.8                | 20.5       | -2.3   | -10%                             | -4.7               | -23%                  | 0.79          |                       |  |            |  |                |  |                                 |  |
| 13                                                                                                                         | 1830"norm'                           | 127.0 | -43.8 | 64.50                 | -1.8 | 34.00               | 28.00    | 8.5    | 35.00   | 1.033                       | 21.5                 | 21.46                     | 21.80               | 19.2       | 0.0    | 0%                               | -2.4               | -11%                  | 0.80          |                       |  |            |  |                |  |                                 |  |
| 14                                                                                                                         | ♀yng adult'                          | 123.0 | -47.8 | 69.00                 | 5.5  | 36.00               | 24.00    | 6.9    | 37.00   | 1.040                       | 20.9                 | 18.16                     | 17.8                | 14.7       | -2.7   | -13%                             | -5.1               | -28%                  | 0.65          |                       |  |            |  |                |  |                                 |  |
| 15                                                                                                                         | ♂fit 40\50                           | 165.0 | -5.8  | 69.00                 | 5.5  | 36.00               | 34.50    | 8.5    | 38.00   | 1.044                       | 20.6                 | 24.36                     | 23.9                | 23.5       | 3.8    | 18%                              | 1.4                | 6%                    | 0.91          |                       |  |            |  |                |  |                                 |  |
| 16                                                                                                                         | ♂yng adult                           | 142.0 | -28.8 | 73.00                 | 4.0  | 35.00               | 28.00    | 8.5    | 35.00   | 1.054                       | 19.8                 | 18.73                     | 17.9                | 14.8       | -1.1   | -6%                              | -3.5               | -19%                  | 0.80          |                       |  |            |  |                |  |                                 |  |
| 17                                                                                                                         | ♀tall model                          | 125.0 | -45.8 | 70.00                 | 6.5  | 35.00               | 25.00    | 7.9    | 34.00   | 1.074                       | 18.3                 | 17.93                     | 17.5                | 13.9       | -0.4   | -2%                              | -2.8               | -15%                  | 0.74          |                       |  |            |  |                |  |                                 |  |
| 18                                                                                                                         | ♀fitns mod                           | 140.0 | -30.8 | 63.00                 | -0.5 | 33.00               | 27.00    | 8.6    | 38.00   | 1.084                       | 17.5                 | 24.80                     | 25.5                | 23.5       | 7.3    | 41%                              | 4.9                | 20%                   | 0.71          |                       |  |            |  |                |  |                                 |  |
| 19                                                                                                                         | ♂fitns mod <sup>1</sup>              | 180.0 | 9.2   | 67.00                 | 3.5  | 33.00               | 34.00    | 9.8    | 38.00   | 1.120                       | 14.8                 | 28.19                     | 28.1                | 27.8       | 13.4   | 90%                              | 11.0               | 39%                   | 0.89          |                       |  |            |  |                |  |                                 |  |
| 20                                                                                                                         | ♀♂anorex                             | 122.5 | -48.3 | 68.00                 | 6.5  | 34.00               | 23.00    | 7.9    | 34.00   | 1.130                       | 14.1                 | 18.62                     | 18.4                | 14.3       | 4.6    | 32%                              | 2.2                | 12%                   | 0.68          |                       |  |            |  |                |  |                                 |  |
| Inputs measured or extrapolated per available data for illustration†                                                       |                                      |       |       |                       |      |                     |          |        |         | ↑20 averaged: 0.985         |                      | 25.1                      | 27.2                | 27.0       | 30.0   | 2.1                              | 10%                | -0.3                  | -4%           | 0.88                  |  |            |  |                |  |                                 |  |
| Note: average BDI v BMI difference of 2.1 (10%) shows Quintelet is relatively 'accurate' even for a "small country" of 20! |                                      |       |       |                       |      |                     |          |        |         | 0.985                       | xtrpNASA density avg |                           | Max variances: -6.0 |            | 13.4   | -8.4                             | 11.0               | 0.38 > 8              |               |                       |  |            |  |                |  |                                 |  |

Illustrative data are in black or extrapolated in green. Reference CDC averages are in gray; waist:hips ratios at far right and Max variance >0.80. “New BMI”=1.3w/h<sup>2.5</sup> is still based only on *w* & *h*. The BMI-BDI<sub>n</sub> “differ” and “BMI<sub>VTL</sub>” columns and, across the bottom, their averages or descended min/maxima compare performance of BDI<sub>n</sub> v. BMI in Fig.1. The BMI formula, invented in 1832, specified it was appropriate only averaging a population, but not for individuals; BDI<sub>n</sub> is more appropriate for individuals by accurately estimating body volume and density.

Superscript note 1 high waist 47.5in, low waist 44.5in  
 2 revealed low bone density prior to a DEXA scan  
 3 near obese but normal as #8 after losing 7lb  
 4 does she appear underweight in image on p1?  
 5 underweight in BDI<sub>n</sub>, not near obese in BMI

#1 (at right in images on p1) ~#4 are at risk due to obesity or morbid obesity.  
 #3 is tall with a trunk maximum of 47.5in above his waist, but a minimum of 44.5in. Although not obese per a BMI of 28.9, he is at risk 6 points higher in BDI<sub>n</sub>.  
 #5 & #8 is the author at 215lb and 208lb mid-diet, a 1-point drop in BMI, but 3+ points lower BDI<sub>n</sub> with an inch reduced waist, depth, & hips. Most loss is in his trunk, equal to 3.2 liters of H<sub>2</sub>O, equating to 7lb weight lost. (His goal is <200.)  
 #6, #7, & #10 are not individuals, but CDC-average American women & men for reference. Females remain borderline obese in BMI and BDI<sub>n</sub> while males in BDI<sub>n</sub> reclassify to just overweight.

#9 is Normal in both BMI & BDI<sub>n</sub> although as a fit 40\50yo she may classify as slightly underweight if normal for females were established higher than for males.  
 #11 is a medium-build senior male who registers underweight in BMI but normal in BDI<sub>n</sub>, so is ill-advised to put on weight.  
 #12's normal BMI is low for her weight & petite figure, her higher BDI<sub>n</sub> inviting suspicion of a bone deficiency prior to diagnosis by a DEXA-scan.  
 #13 is Quetelet's 1830 "normal." #14 is the female on p1 is underweight in BMI, but is healthy in BDI<sub>n</sub>. #15 is near overweight by BMI, but is normal in BDI<sub>n</sub>.

Scoring less underweight in  $BDI_n$  than in BMI, #16 is a typically slender young adult male, and #17 a runway model-like female. Fit athletes often score higher in BMI, yet female fitness model #18 is normal in BMI, but underweight in  $BDI_n$ .

#19 is borderline obese in BMI, but registers severely underweight in  $BDI_n$ . We deem #20 nearly OK in BMI, but is dangerously underweight in  $BDI_n$ .

### Propagation of uncertainty, error compensation, other variables

Table.1's column " $BMI_{TL}$ " has erratic errors due to its formula. For a different reason,  $BDI_n$  could err as much as 0.15ℓ (~4.5oz of fat) due to sloppy measuring, prescribed in Appx.A. Off by ½ inch (~1cm) in any three of the five size measures can cause this error.

$BDI_n$ 's volume model relies on assumptions with errors deemed small cf. overall error. E.g. average head density is ~1.1g/cm<sup>3</sup>, its volume equivalent to ~10% larger.<sup>o</sup> <sup>19</sup> Bodily features beyond the trunk in Fig.2L may not entirely fill voids above the shoulders. To compensate,  $\Delta \approx 0.925$  brings the average BD to the accepted 0.985kg/ℓ for an ~7.5% smaller  $V_{trunk}$  to complete expression [18].

We carry math to at least three significant digits after rounding, four digits if a "1" is leading. In some expressions we use "≈" for "approximately equal." With only 20 subjects, Table.1 & Fig.1 are sensitive to each added dataset. Statistical analysis of variance (ANOVA) of a range of subjects (≥601) would settle variables **M**, **N**, &  $\Delta$  as constants. Simple clinical trials could provide that data.

### Discussion & further work

In the 1830s, one 5ft 4in weighing 127 lb with >1.0 g/cm<sup>3</sup> body density is petite & lean cf. Americans today, whose ~0.985 g/cm<sup>3</sup> mean reveals fat has overtaken muscle. One need only visit an American mall to observe all sizes & shapes of women & men: wannabe runway models defying  $h_{sit} \approx L_{leg}$  vs. long-torso-short-legged individuals. Ratios of waist-to-hips much higher than the desirable 0.7~0.9 for females to males varying by cultural norms (calculated from  $BDI_n$  data in Table.1 under column "pw:ph").

Ignoring salient body differences, BMI misleads re adiposity. Fat weighs less than muscle: a fat person floats on water while a lean person sinks. Yet as the proportion of muscle rises, BMI rises.<sup>p</sup> Note those individuals in Table.1 who are mis-categorized in BMI: five as borderline obese who are not, two as healthy who are severely underweight.<sup>q</sup> In Appx.D, other indices are plotted against the density BD that renders  $BDI_n$  linear. In Fig.3, more data for this paper's  $V_{body}$  may slightly re-slope (adjust sensitivity) of  $BDI_n$ 's trendline, but will not render other indices less erratic.<sup>20</sup>

New sets of adiposity categories have been proposed for BMI, standardized in 1998 by global authorities: nine suspiciously round-numbered from "severely underweight" to three classes of

obesity (except for two *band-aid* Asian categories) that do not present adiposity credibly.  $BDI_n$ 's higher precision may warrant more nuanced categories by gender, race, & age (adding children & seniors) that are easily implemented by check boxes in an app. An app could be agnostic to round numbers. Should a consensus arise for more objective boundaries, the scalable form  $BDI_x$  prior to normalizing needs only rescaling M & N. Changes in categories do not diminish  $BDI_x$  or  $BDI_n$  as the successor metric of adiposity.

Optimized categories or rescaling  $BDI_n$  from the scalable form  $BDI_x$  could include children or those short of stature from illness, dwarfism, or Down's Syndrome for whom  $(arm-span + L_{leg})/h =$  about 2.7  $h_{sit}$  that is about 10% lower than that ratio for adults.<sup>r</sup>

Several interviewees stressed that being told their BMI was emotionally devastating to their self-image. A truer  $BDI_n$  could ameliorate these reactions, and help patients to focus undistracted on heeding physicians' advice about their health.

### Conclusions – should $BDI_n$ replace BMI?

To track the public health of Belgium, Quetelet in 1832 devised a brilliant statistical axiom  $w/h^2$ . It worked because averaging large numbers smooths its errors. Quetelet ruled out use for individuals, as for each the proper indicator of individual fatness v. leanness is body density, *weight divided by volume*,  $w/V$ . Yet in 1972, Ancel Keys hijacked Quetelet's formula, re-naming it "BMI" for an individual obesity index to escalate life insurance rates!

Soon BMI was adopted uncritically by the global healthcare community, who read a patient's obesity from a chart. Imposing an arithmetic average on a person regardless of leanness has been criticized for 50+years as a gross miscarriage of medical science. Finally BMI has been ill-advised by the AMA and NIH...

"Use of BMI as a measure of adiposity and predictor of health risks (cardiovascular, stroke, type 2 diabetes, cancer) requires understanding its limitations for an individual... BMI [exhibits] an appreciable variance (60–70%) in measured fatness in groups of adults... an unreliable indicator of the body composition of an individual."<sup>21</sup> In general varying in bone & muscle, women, children, and non-white races have no "Normal" enshrined, imposing on them in error the Normal 21.5 of a white male.

Fig.1 illustrates 20 "individuals" in Table.1 whose BMIs careen erratically –8.4 points below to +11.0 above BMI's own trendline, where they ought to lie. Ignoring body density to indicate the adiposity, BMI ignores shape, age, gender, and race. Ex.1 & 2 show that any two individuals identical in weight & height will have the same BMI despite who's fat and who's lean. **Orange BMI data** points in Fig.1 whiplash by individual body density.

<sup>o</sup> Barber, Brockway, Higgins 1970-"The density of tissues in and about the head."

<sup>p</sup> "You're Overweight. Or are you?" [https://www.wsj.com/articles/youre-overweight-or-are-you-11628847001?st=16n04rqatqgnqgo&reflink=article\\_copyURL\\_share](https://www.wsj.com/articles/youre-overweight-or-are-you-11628847001?st=16n04rqatqgnqgo&reflink=article_copyURL_share) - WSJ

<sup>q</sup> Scaled to 5'9" 110lbs, Barbie's BMI is 16; adding waist 18in & hips 33,  $BDI_n$  is 9!

<sup>r</sup> [https://www.researchgate.net/publication/5462786\\_Auxology\\_Is\\_a\\_Valuable\\_Instrument\\_for\\_the\\_Clinical\\_Diagnosis\\_of\\_SHOX\\_Haploinsufficiency\\_in\\_School-Age\\_Children\\_with\\_Unexplained\\_Short\\_Stature/](https://www.researchgate.net/publication/5462786_Auxology_Is_a_Valuable_Instrument_for_the_Clinical_Diagnosis_of_SHOX_Haploinsufficiency_in_School-Age_Children_with_Unexplained_Short_Stature/)

*BDI<sub>n</sub> by design is inclusive of all individuals.* BDI<sub>n</sub> is based on a bio-engineering model of *individual body volume* using only four additional measurements that with practice take only one minute more to make. The model may not be perfect, only better than BMI's using only one measurement for volume. From the more accurate volume we find a scalable body density index BDI<sub>x</sub> to fit optimal categories. For mainstream acceptance, we normalize BDI<sub>x</sub> to BMI's familiar categories and sensitivity slope as BDI<sub>n</sub>.

With data gathered per **Appx.A** and calculated online, BDI<sub>n</sub> translates as better clinical health assessment and advice re weight-related risks, and with fewer unintended mental health consequences for patients distracted from that advice by anxiety over body image. A full-featured online calculator app is free to peruse, or for clinicians to submit data after in-app request to.<sup>5</sup>

The precision of BDI<sub>n</sub> depends on clinical data posted in the spreadsheet online. Greater precision may be unnecessary, and entails more measurements that are awkward for patient and clinician, e.g. caliper measures of hip depth or leg breadth.

For most individuals, BMI distorts body fatness between 13.4 points too high to –6 too low. However BDI<sub>n</sub> normalized to BMI categories is linear, undistorted for any individual body shape and leanness, as illustrated even with limited data in *Fig.1* & *Table.1*. Evaluated alongside BDI<sub>n</sub>, it is apparent that BMI is unsuitable to continue in clinical use as a reliable measure of adiposity, and that this paper's proposed BDI<sub>n</sub> is a suitable successor.

#### Appendix A – BDI<sub>n</sub> measurement method & calculation

This paper's model in *Fig.2 L* calculates *V<sub>body</sub>* from five (5) size measures: overall *height h*, plus *sitting height h<sub>sit</sub>*, *waist depth d*, and perimeters *waist p<sub>w</sub>* and *hips p<sub>h</sub>* that with practice take only one minute more to measure. BDI<sub>n</sub> results are only as good as inputs to the nearest ¼in (½cm) and ½lb (¼kg). *Do not use clothing sizes that are typically smaller than real sizes.* Have the subject to relax (no sucking-in). The six BDI<sub>n</sub> measurements are:

- w** weight (same as BMI) to the nearest ±0.5lb (0.25kg);
- h** standing height (same as BMI) to the nearest 0.25in (0.5cm);
- h<sub>sit</sub>** sitting height ("crown to rump" seated on a firm surface, with feet suspended for no pressure) to the nearest ±¼in or ½cm;
- p<sub>w</sub>** waist perimeter (around small of back, standing) ±0.25in (0.5cm);
- d** depth of trunk (from small of the back, standing) ±0.25in (0.5cm);<sup>t</sup>
- p<sub>h</sub>** hips perimeter (around rump's widest, standing) ±0.25in (0.5cm).

<sup>5</sup>[https://docs.google.com/spreadsheets/d/1lWthOCmxGyl5mqL7epKoFLruWMsySq\\_JjC\\_0Zlq1ol/](https://docs.google.com/spreadsheets/d/1lWthOCmxGyl5mqL7epKoFLruWMsySq_JjC_0Zlq1ol/)

<sup>t</sup> By body caliper, or solid wire bent to an Ω-shape, tightened to graze the skin.

<sup>u</sup> Scaling variables M, N, & Δ become constants with more datasets.

<sup>v</sup> <https://www.cdc.gov/nchs/fastats/body-measurements.htm> – cf. *Table.1*.

Enter six (6) data and the units constant **C** (1,000 for metric or 27.68 for Imperial) in the online calculator,<sup>5</sup> or copy & paste the **BDI<sub>n</sub> expression (19)** in any online search field, e.g. Google, or in a spreadsheet cell. Substitute (select precisely & overtype, no spaces) all *measured inputs in bold italics* in the same units as **C**, and press Enter.<sup>u</sup> The calculated result is **BDI<sub>n</sub>** =

$$=13.12+76.4*(1.142-C*w/((0.925*h_{sit}*3.14*d*((0.1904*p_h+(p_w^2/(2*3.14^2)-(d/2)^2)^{0.5}))/4)+0.0277*p_h^2*(h-h_{sit}))) \quad (\text{index normalized to BMI}) \quad [18]$$

To compare indices, similarly paste in a search field or spreadsheet cell:  
**BMI<sub>metric</sub>=w/h<sup>2</sup>** or **BMI<sub>imperial</sub>=703\*w/h<sup>2</sup>**; **'New BMI'<sub>metric</sub>=1.3\*w/h<sup>2.5</sup>**.

#### Appendix B – CDC 2021 average *h*, *w*, & *p<sub>w</sub>* of Americans ≥20yr<sup>v</sup>

|                                        |                                        |                                       |                   |
|----------------------------------------|----------------------------------------|---------------------------------------|-------------------|
| Height <b>h</b>                        | Men: 69.0                              | Women: 63.5                           | Overall: 66.25 in |
| Weight <b>w</b>                        | 199.8                                  | 170.8                                 | 185.3 lb          |
| <b>*BMI=29.7, BDI<sub>n</sub>=22.8</b> | <b>*BMI=29.8, BDI<sub>n</sub>=25.3</b> | <b>BMI=29.5, BDI<sub>n</sub>=25.4</b> |                   |
| Waist <b>p<sub>w</sub></b>             | 40.5                                   | 38.7                                  | 39.6 in           |

<sup>\*</sup>Prior 2015–18 ranges: US adult males BMI 24.2–30.1; females BMI 24.2–32.9. By 2021 males on average became 4% heavier and females 6%, all up a full point in BMI – [https://www.cdc.gov/nchs/data/series/sr\\_03/sr03-046-508.pdf](https://www.cdc.gov/nchs/data/series/sr_03/sr03-046-508.pdf)

Health body compositions are on p1 for the average American male & female. Many differences by race are not included.

#### Appendix C – body volume *V<sub>body</sub>* model calculation<sup>w</sup>

*V<sub>body</sub>* is modeled in *Fig.2L* as an elliptical cylinder and truncated cones for legs. Ellipses (or special case a circle of a rotund body) have semi-minor & semi-major axes *a* & *b* and area *A*=π*ab*. Then

$$V_{trunk1st} \approx \text{Area}_{base} * h_{sit} \approx \pi a_{trnk} b_{trnk} h_{sit} \approx \pi (d/2) b_{trnk} h_{sit} \quad m^3 \quad [5]$$

where **a<sub>trnk</sub>** = **d/2**, the measured trunk depth, *h<sub>sit</sub>* is sitting height, and *b<sub>trnk</sub>* calculated next from waist & hips perimeters *p<sub>w</sub>* & *p<sub>h</sub>*.

NASA published human factors for 40yr-old Americans in 2000.<sup>x</sup> The 50<sup>th</sup> percentile of hip breadth, **2\*b<sub>h</sub>** in *Fig.2L*, averages waist depth **d**=0.6510\*2*b<sub>h</sub>* for men, 0.6083 for women,<sup>14</sup> each within ~4% of their average of **0.630\*2*b<sub>h</sub>***. Hips in typical cross-section are determined by hipbone geometry, modelled by an ellipse of ratio 0.630, **a<sub>h</sub>**: **b<sub>h</sub>** (*a<sub>h</sub>* not shown), dimensioned by hips perimeter *p<sub>h</sub>*. We find *b<sub>h</sub>* from *p<sub>h</sub>*. Then 0.630*b<sub>h</sub>* substitutes for *a<sub>h</sub>*.

The relationship between *a* & *b* and perimeter *p* within a 2% error is the root mean square (RMS)  $p=2\pi\sqrt{(a^2+b^2)/2}$ , then

$$b_h^2=p_h^2/2\pi^2-0.63^2b_h^2, \quad 1.3969b_h^2=p_h^2/2\pi^2, \quad b_h=0.1904p_h \quad m \quad [6] \quad ^v$$

Unlike the hips ellipse, the waist of perimeter *p<sub>w</sub>* is an ellipse where **a<sub>w</sub>** = **a<sub>trnk</sub>** = **d/2** (half measured trunk depth), ranging from a flat tummy to an obese circle. The waist semi-major axis

<sup>w</sup> For continuity formulae numbers continue after [4] page 2 and resume on p3.

<sup>x</sup> <https://msis.jsc.nasa.gov/sections/section03.htm> – The US mean age is 38yr. Note NASA's typo "Japanese female" titling the chart for American female. Other anthropometric data at - <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2872302/>

<sup>v</sup>  $p=2\pi\sqrt{(a^2+b^2)/2}$ ,  $p^2=2\pi^2(a^2+b^2) = 2\pi^2a^2+2\pi^2b^2$ , then  $b^2=p^2/2\pi^2-a^2$ .

$$b_w \approx \sqrt{(p_w^2/2\pi^2 - a_w^2)} \approx \sqrt{(p_w^2/2\pi^2 - (d/2)^2)} \quad m \quad [7].$$

Fig.2L shows a trunk volume of  $h_{sit}$  times the area of an ellipse that we imagine by averaging waist & hips (either one larger). Substituting in [5] a trunk half-breadth average  $b_{trnk} = (b_h + b_w)/2$

$$\begin{aligned} V_{trunk1st} &\approx \pi(d/2)b_{trnk}h_{sit} \approx h_{sit}\pi(d/2)(b_h+b_w)/2 \\ &\approx \frac{1}{4}\Delta h_{sit}\pi d(0.1904p_h + \sqrt{(p_w^2/2\pi^2 - (d/2)^2)}) \quad m^3 \quad [8].^z \end{aligned}$$

We complete this 1<sup>st</sup> approximation of trunk volume in the Errors...uncertainties section to correct by a factor  $\Delta$  for voids not filled by other trunk features vs. the head's equivalent volume.

With better nutrition and less disease, we've evolved longer-legged than da Vinci's *Vitruvian Man*. We define leg height *not* by the joints in Fig.2R, but standing height  $h$  minus  $h_{sit}$  that extends from below the hip joints.<sup>aa</sup> A first approximation of a leg volume is  $\frac{1}{3}L_{leg}$  times the cone base area of radius  $b_h/2$ , so  $r_{leg} = 0.0952p_h$ .

$$V_{leg1st} \approx \frac{1}{3}A \cdot L_{leg} \approx \frac{1}{3}\pi r^2(h - h_{sit}) \approx \frac{1}{3}\pi(0.0952p_h)^2(h - h_{sit}) \quad m^3 \quad [9].$$

Then the 1<sup>st</sup> approximation of the volume of two legs simplifies to

$$V_{legs(2)1st} \approx \frac{2}{3}\pi 0.00906p_h^2(h - h_{sit}) \approx 0.018981p_h^2(h - h_{sit}) \quad m^3 \quad [10].$$

Refining the model, in Fig.2L the dancer is *en pointe*, but unsquashed, as in zero gravity. Foot length averages  $\sim 0.1515h$ ,<sup>22</sup> or  $\sim 0.3030L_{leg}$  for  $L_{leg} \sim h/2$ . By Fig.2R, the ankle joint is above the floor at  $0.039h$ , or  $\sim 0.078L_{leg}$ .  $L_{leg}$  elongates toes to virtually below the floor by  $0.3030 - 0.078 \approx 0.225$ .<sup>23</sup> So the 1<sup>st</sup> of 3 adjustment coefficients applied to  $L_{leg}$  is  $1.000 + 0.225 \approx 1.225L_{leg}$ .

By observation, the dancers' toes do not come to a point, but truncate the leg cone visualized as a circle of diameter  $\sim b/5$  after extending  $L_{leg}$  farther by  $\sim 1/5$ . The truncation area is a squared fraction of the base cone, or  $1^2/5^2 = 1/25$ ; the truncation volume a cubed ratio  $\sim 1^3/5^3 = 1/125$  or a factor of  $\sim 0.992$ . The three leg cone refinements combine to  $1.225 \cdot 1.20 \cdot 0.992 = 1.458$  as in

$$V_{legs(2)} \approx 0.01898p_h^2 \cdot L_{leg} \cdot 1.458 \approx 0.0277p_h^2(h - h_{sit}) \quad m^3 \quad [11].$$

From [4] with **inputs in bold** and  $\Delta$  a volume model correction factor described later, the combined  $V_{trunk} + V_{legs(2)} = V_{body}$

$$\approx \Delta h_{sit}\pi d(0.1904p_h + \sqrt{(p_w^2/2\pi^2 - (d/2)^2)})/4 + 0.0277p_h^2(h - h_{sit}) \quad m^3 \quad [12].$$

This seemingly daunting expression is simple for a computer to determine individual Body Density BD, continuing on page 3.

#### Appendix D – other BMI replacements: “New BMI” and “BFATI”

Others have proposed improving upon BMI (“BFATI” or “New BMI” in Fig.3), or perfecting the measure of body density (Mayo Clinic's body volume scanner. A viable argument by University of Rochester's Holly Russell MD is not to *measure* at all, that the best evaluator of a patient's “ideal weight” may be a physician's expert observation. In the overline on p1, the author's GP said:

<sup>z</sup>  $V_{trnk} \approx h_{sit}\pi(d/2)^2(b_h+b_w)/2 \approx \frac{1}{4}h_{sit}\pi d^2(b_h+b_w)$ .

<sup>aa</sup> Sitting height  $h_{sit}$  measured per Appx A; leg length (subischial)  $L_{leg} = h - h_{sit}$ .

“Your BMI is borderline obese, but you don't look obese!”

Visible muscle definition in extremities & abdomen may in many cases be a more accurate indicator of adiposity than BMI! A worthy study would be to compare observations with BDI<sub>n</sub>.

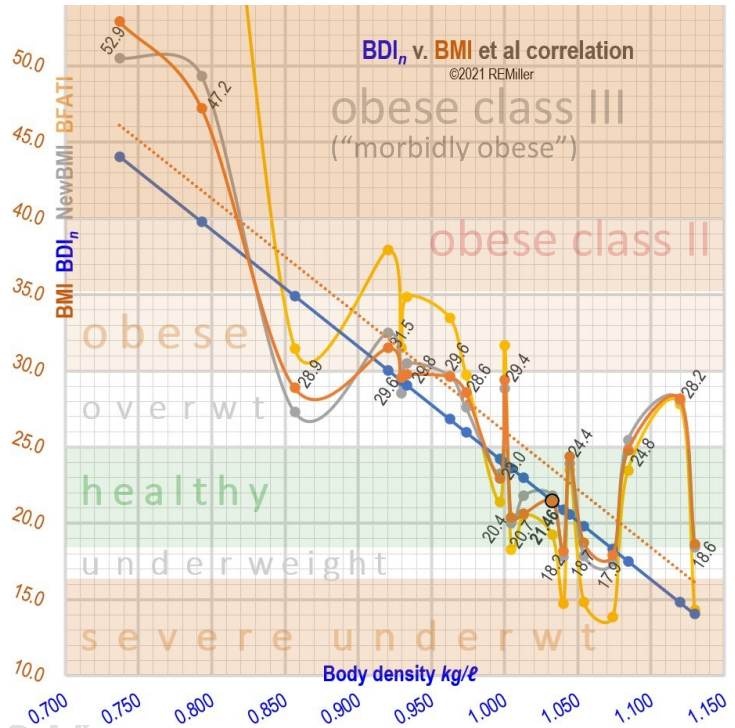


Fig.3 – Compared to this paper's BDI<sub>n</sub>, “New BMI” and “BFATI” are as erratic by individual as BMI – all are based on similar formulae that miscalculate density.

#### The author, editors, & auditors

An internationally recognized engineer, graduate school professor, and Peabody winning filmmaker, **Robert E (Robin) Miller III** has communicated health issues from cardiac catheterization methods to physical & mental disabilities.<sup>bb</sup> He's presented research to symposia in the US, Canada, Germany, Austria, The Netherlands, and Italy. A protégé of an inventor of the ENIAC computer, as Filmmaker Technology he works in systems design, integration, & publication at [www.filmaker.com](http://www.filmaker.com). In this paper he solves a biomedical engineering problem with an applied scientist's sense of what we can economically model and implement. <sup>A</sup>

Editor **Yehia Y Mishriki MD** is a widely published clinician and educator specializing in internal medicine. Now Emeritus Professor of Medicine of the Lehigh Valley Health Network, his career spanned decades with Eastern Pennsylvania's largest healthcare provider. He's cited frequently having authored more than one hundred scholarly works on medical science, diagnosis, and treatment.

Editor **Gilbert H Aykroyd BSEE** has served PBS national committees, the Society of Motion Picture & Television Engineers (SMPTE), and Arizona Society of Broadcast Engineers (SBE). As CTO of #9 market KAET-TV8, he supervised construction and moving operations to a 6-story building in downtown Phoenix AZ, and converting both TV8 and WLVT-TV39, Allentown PA, to digital High Definition.

Peer auditors: **Terrill E Theman MD** & bio-engineering prof, Lehigh Univ; thoracic surgeon, retired; **James J Boylan MD**, **John S Kintzer MD** St Lukes Univ. Health, **Rena M Lambert DO** obesity specialist, Lehigh Valley/Jefferson Hospital.

<sup>bb</sup> Seminar for 200 cardiologists by Fayaz A Schall MD televised from Washington Adventist Hosp., Alzheimer's therapies, adaptations for persons with disabilities, and pioneering use of amniocentesis (winner of Corporation for Public Broadcasting “Outstanding Documentary”).

**Two upon whose shoulders we stand** (quoting other researchers)... “A social sciences founder, **Adolphe Quetelet’s** (1796–1874) 1835 ‘Treatise of Man and the Development of his Faculties’ was hailed ‘one of the greatest books of the 19th century.’”<sup>24</sup> A tireless promoter of statistical data collection and analysis, Quetelet organized in 1853 the first International Statistical Congress.” More than a century later in 1972, Quetelet’s brilliant 1832 analysis tracking of a nation’s health was renamed “BMI” by Ancel Keys, below. Note in Table.1 that when averaged even over a ‘very small country’s’ population of 20, Quetelet’s Formula becomes accurate within 2.1 points (10%) of  $BDI_n$  based on individual body density, a testament to his statistical approach invoking ‘the law of large numbers.’

“Physiologist **Ancel Keys**’ (1904–2004, developer of K-rations) ‘Seven Countries Study’ culminated 20+ years’ effort to discredit MetLife’s weight-for-height ratio as a measure of obesity in its actuarial tables.<sup>25</sup> In the study, Keys coined Quetelet’s formula “BMI,” universally used for 50+ years since, leading to fatness categories established in 1995 by the WHO, but today ill-advised by the AMA and the NIH. Keys’ study drew criticism for including only males, deleting data that didn’t suit his hypothesis, and failing in its claim that BMI was a predictor of heart disease.

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